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VELOCITY STATISTICS OF ROUND, FULLY-DEVELOPED, BUOYANT TURBULENT PLUMES

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Abstract

An experimental study of the structure of round buoyant turbulent plumes was carried out, limited to conditions within the fully developed (self-preserving) portion of the flow. Plume conditions were simulated using dense gas sources (carbon dioxide and sulfur hexafluoride) in a still air environment. Velocity statistics were measured using laser velocimetry in order to supplement earlier measurements of mixture fraction statistics using laser-induced iodine fluorescence. Similar to the earlier observations of mixture fraction statistics, self-preserving behavior was observed for velocity statistics over the present test range (87-151 source diameters and 12-43 Morton length scales from the source), which was farther from the source than most earlier measurements. Additionally, the new measurements indicated that self-preserving plumes are narrower, with larger mean streamwise velocities near the axis (when appropriately scaled) and with smaller entrainment rates, than previously thought. Velocity statistics reported include mean and fluctuating velocities, temporal power spectra, temporal and spatial integral scales and Reynolds stresses.

Nomenclature

B_o	=	source buoyancy flux, Eq. (3)
d	=	source diameter
$E_i(n)$	=	temporal power spectral density of velocity component i
E_o	=	entrainment constant, Eq. (19)
f	=	mixture fraction
$F(r/(x-x_o))$	=	scaled radial distribution of $\bar{f}$ in self-preserving region
Fr_o	=	source Froude number, Eq. (5)
g	=	acceleration of gravity
k_f, k_u	=	plume width coefficients based on $\bar{f}$ and $\bar{u}$
ℓ_f, ℓ_u	=	characteristic plume radii based on $\bar{f}$ and $\bar{u}$
ℓ_M	=	Morton length scale, Eqs. (1) and (4)
M_o	=	source specific momentum flux, Eq. (2)
n	=	frequency
Q	=	plume volumetric flow rate
r	=	radial distance
Re_o	=	source Reynolds number, $u_o d/\nu_o$
u	=	streamwise velocity
$U(r/(x-x_o))$	=	scaled radial distribution of $\bar{u}$ in self-preserving region
v	=	radial velocity
w	=	tangential velocity
x	=	streamwise distance
η	=	dimensionless radial distance, Eq. (15)
ν	=	kinematic viscosity
ρ	=	density
τ_f, τ_u	=	temporal integral scales of f' and u'

Subscripts

c	=	centerline value
o	=	initial value or virtual origin location
∞	=	ambient value

Superscripts

$(\bar{\ })$	=	time-averaged mean value
$(\bar{\ })'$	=	root-mean-squared fluctuating value

Introduction

The structure of round buoyant turbulent plumes in still and unstratified environments is an important fundamental problem that has attracted significant attention since the classical study of Rouse et al. (1952). However, recent work has highlighted the need for more information about buoyant turbulent plumes in order to address effects of turbulence/radiation interactions (Kounalakis et al., 1991), and to help benchmark models of buoyant turbulent flows (Dai et al., 1994). Thus, the objective of the present investigation was to measure mean and fluctuating velocity properties in round buoyant turbulent plumes, in order to supplement earlier measurements of mean and fluctuating scalar properties (mixture fractions) in these flows, due to Dai et al. (1994). The fully-developed region, where effects of the source have been lost and the properties of the flow become self preserving, was emphasized due to its fundamental importance for simplifying both theoretical considerations and the interpretation of the measurements (Tennekes and Lumley, 1971), even though few practical plumes reach these conditions.

Several reviews of turbulent plumes have appeared (Chen and Rodi, 1980; Kotsovinos, 1985; List, 1982; Papanicolaou and List, 1987, 1988); therefore, the discussion of past studies will be brief. The earliest work emphasized the development of similarity relationships for flow properties within fully-developed (self-preserving) buoyant turbulent plumes (Rouse et al., 1952; Morton et al., 1956; Morton, 1959). Subsequently, many workers reported observations of mean properties at self-preserving conditions, however, the various determinations of centerline values and flow widths generally were not in good agreement (Abraham, 1960; Chen and Rodi, 1980; Dai et al., 1994; George et al, 1977; Kotsovinos, 1985; Kotsovinos and List, 1987; Mizushima et al., 1979; Nakagome and Hirata, 1977; Ogino et al., 1980; Papanicolaou and List, 1987, 1988; Papantoniou and List, 1989; Peterson and Bayazitoglu, 1980; Seban and Behnia, 1976; Shabbir and George, 1992; Zimin and Frik, 1977). Papanicolaou and List (1987, 1988), Papantoniou and List (1989) and Dai et al. (1994) attribute these discrepancies mainly to problems of fully reaching self-preserving conditions, with conventional experimental uncertainties serving as a contributing factor.

Self-preserving round buoyant turbulent plume conditions are reached when streamwise distances from the plume source are large in comparison to two characteristic length scales, as follows: (1) the source diameter, as a measure of conditions where effects of source disturbances have been lost; and (2) the Morton length scale, as a measure of conditions when the buoyant features of the flow are dominant. For general buoyant jet sources, the Morton length scale is defined as follows (Morton, 1959; List, 1982; Papanicolaou and List, 1988):

$$\ell_M = M_o^{3/4} / B_o^{1/2} \qquad (1)$$

For round plumes with uniform properties defined at the source (similar to the present experiments), the source specific momentum flux, M_o, and the source buoyancy flux, B_o, are defined as follows (List, 1982; Dai et al, 1994):

$$M_o = (\pi/4)d^2u_o^2 \qquad (2)$$

$$B_o = (\pi/4)d^2u_o g\,|\rho_o-\rho_\infty|\,/\rho_\infty \qquad (3)$$

where an absolute value of the density difference has been used in Eq. (3) to account for both rising and falling plumes. Substituting Eqs. (2) and (3) into Eq. (1) then yields the following expression for ℓ_M for round plumes that have uniform properties at the source:

$$\ell_M = (\pi/4)^{1/4}(\rho_\infty d u_o^2/(g|\rho_o-\rho_\infty|))^{1/2} \qquad (4)$$

The ratio, ℓ_M/d, is proportional to the source Froude number, defined as follows for conditions analogous to those of Eq. (4) (List, 1982):

$$Fr_o = (4/\pi)^{1/4}\,\ell_M/d = (\rho_o u_o^2/(g|\rho_\infty-\rho_\infty|d))^{1/2} \qquad (5)$$

The source Froude number is a convenient measure of the dominance of buoyancy at the source, e.g., $Fr_o = 0$ and ∞ for purely buoyant and for purely nonbuoyant sources, respectively.

Papanicolaou and List (1987, 1988) suggest that buoyancy-dominated conditions for mean and fluctuating quantities are reached for $(x-x_o)/\ell_M$ greater than roughly 6 and 14, respectively, which has been satisfied by most past measurements seeking results at self-preserving conditions (Dai et al., 1994). However, aside from the measurements of Papantoniou and List (1989) and Dai et al. (1994), to be discussed subsequently, existing measurements of radial profiles of mean and fluctuating properties in buoyant turbulent plumes have been limited to $(x-x_o)/d$ in the range 6-62, with most measurements emphasizing the lower end of this range, see Papanicolaou and List (1987, 1988), Shabbir and George (1992), George et al. (1977), Ogino et al. (1980), Nakagome and Hirata (1977), and Peterson and Bayazitoglu (1992), among others. This range of normalized streamwise distances is rather small to achieve self-preserving conditions, based on findings for nonbuoyant round turbulent jets where values of $(x-x_o)/d$ greater than roughly 40 and 100 are required to achieve self-preserving profiles of mean and fluctuating properties, respectively (Hinze, 1975; Tennekes and Lumley, 1972). Similar behavior for round buoyant turbulent plumes recently has been established by Papantoniou and List (1989) and Dai et al. (1994). These measurements were limited to scalar properties and found that self-preserving mean and fluctuating mixture fractions (i.e., the mass fraction of source material in a sample) only were achieved at $(x-x_o)/d$ and $(x-x_o)/\ell_M$ greater than roughly 100 and 10, respectively. These results also showed that self-preserving buoyant turbulent plumes were narrower, with larger mean and fluctuating scalar properties at the axis (when appropriately scaled), than earlier reported measurements of self-preserving scalar properties made closer to the source. Finally, it seems likely that self-preserving behavior for other properties only is achieved at comparable conditions.

The preceding discussion suggests that existing measurements of mean and fluctuating velocities within round buoyant turbulent plumes probably involve transitional plumes. Thus, the objective of the present investigation was to extend the scalar property measurements of Papantoniou and List (1989) and Dai et al. (1994) to consider mean and fluctuating velocity properties within the self-preserving region of round buoyant turbulent plumes. Present test conditions were identical to those of Dai et al. (1994), and involved source flows of carbon dioxide and sulfur hexafluoride in still air at room temperature and atmospheric pressure. This approach yields downwardly-flowing negatively-buoyant plumes in still and unstratified environments, and allows straightforward specification of the buoyancy flux within the test plumes.

Experimental Methods

Test Apparatus. Description of the experimental apparatus will be brief, see Dai et al. (1994) for more details. The test plumes were within a screened enclosure (which could be traversed to accommodate rigidly-mounted instrumentation) that was mounted within an outer enclosure. The plume sources were long round tubes that could be traversed in the vertical direction within the inner enclosure for measurements at various streamwise distances from the source. The ambient air within the enclosures was seeded with oil drops (roughly 1 μm nominal diameter) for laser velocimetry (LV) measurements of velocities, using several multiple jet spray generators (TSI, model 9306) that discharged above the screened top of the outer enclosure. In the self-preserving region where present measurements were made, maximum mixture fractions were less than 6%; therefore, effects of concentration bias (because only the ambient air was seeded) were negligible.

Instrumentation. Dual-beam, frequency-shifted LV was used for the velocity measurements, based on the 514.5 nm line of an argon-ion laser. The optical axis of the LV passed horizontally through the flow with signal collection at right angles to the optical axis to yield a measuring volume having a diameter of 400 μm and a length of 260 μm. A darkened enclosure as well as a laser line filter in front of the detector were used to minimize effects of background light. Various orientations of the plane of the laser beams were used to find the three components of mean and fluctuating velocities, as well as the Reynolds stress, as described by Lai and Faeth (1987).

The detector output was amplified and processed using a burst counter signal processor (TSI, model 1980B). The low-pass filtered analog output of the signal processor was sampled at equal time intervals in order to avoid problems of velocity bias, while directional ambiguity and bias were controlled by frequency shifting. The detector output was sampled at rates more than twice the break frequency of the low-pass filter in order to control alias signals. Seeding levels were controlled so that effects of step noise contributed less than 3% to determinations of velocity fluctuations, based on measurements of temporal spectra to be discussed later. Experimental uncertainties (95% confidence) were mainly governed by finite sampling time limitations and are estimated to be less than 5 and 13% for mean and fluctuating velocities, respectively; the corresponding uncertainties for Reynolds stresses are estimated to be less than 16%.

Test Conditions. The experiments involved carbon dioxide and sulfur hexafluoride plumes as summarized in Table 1. For $(x-x_o)/d \geq 87$, where self-preserving conditions were observed, the Kolmogorov microscales of velocity fluctuations were less than 350 μm; therefore, the spatial resolution of present measurements was not sufficient to resolve the smallest scales of turbulence. Present source conditions were identical to those of Dai et al. (1994), however, the locations of the virtual origins of the carbon dioxide plume, based on $\bar{f}$ from Dai et al. (1994) and based on $\bar{u}$ for the present measurements, were not identical. It was beyond the scope of the present investigation to study the reasons for different locations of these virtual origins, however, such behavior is not surprising because the initial conditions and Prandtl/Schmidt numbers are not the same for mixture fractions and velocities.

Self-Preserving Scaling

The general state relationship for density as a function of mixture fraction, assuming ideal gas behavior, is given as follows for the present plume flows (Dai et al., 1994):

$$\rho = \rho_\infty/(1-f(1-\rho_\infty/\rho_o)) \qquad (6)$$

Table 1. Summary of test conditions[a]

Source Properties	CO_2	SF_6
Density (kg/m^3)	1.75	5.87
Kinematic viscosity (mm^2/s)	8.5	2.6
Diameter (mm)	9.7	6.4
Average velocity (m/s)	1.74	1.89
Reynolds number	2000	4600
Froude number	7.80	3.75
Morton length scale, ℓ_M/d	7.34	3.53
Virtual origin, based on $\bar{f}, x_o/d$[b]	12.7	0.0
Virtual origin, based on $\bar{u}, x_o/d$	0.0	0.0

[a]Flow directed vertically downward in still air with an ambient pressure, temperature, density and kinematic viscosity of 99 ± 0.5 kPa, 297 ± 0.5 K, 1.16 kg/m^3 and 14.8 mm^2/s. Source passage length-to-diameter ratios of 50:1.
[b]Based on the measurements of Dai et al. (1993).

Then, noting that $f << 1$ in the self-preserving region, Eq. (6) can be linearized as follows:

$$\rho = \rho_\infty + f\rho_\infty(1-\rho_\infty/\rho_o), f << 1 \quad (7)$$

Under present assumptions, conservation principles and the state relationship for density imply that the buoyancy flux is conserved for buoyant turbulent plumes. Then mean streamwise velocities and mixture fractions can be scaled as follows in the self-preserving region, where flow properties are independent of source properties like d and u_o (List, 1982):

$$\bar{u}((x-x_o)/B_o)^{1/3} = U(r/(x-x_o)) \quad (8)$$

$$\bar{f}gB_o^{-2/3}(x-x_o)^{-5/3}|d\ln\rho/df|_{f\to 0} = F(r/(x-x_o)) \quad (9)$$

For present conditions, it can be seen from Eq. (7) that:

$$|d\ln\rho/df|_{f\to 0} = |\rho_o - \rho_\infty|/\rho_o \quad (10)$$

is a measure of the buoyancy potential with the extent of mixing. As before, an absolute value has been used in Eq. (10) to account for both rising and falling plumes. The x_o in Eqs. (8) and (9) are the virtual origins for $\bar{u}$ and $\bar{f}$, respectively, as noted in Table 1. The $U(r/(x-x_o))$ and $F(r/(x-x_o))$ are appropriately scaled radial profile functions of mean streamwise velocities and mixture fractions, which become universal functions in the self-preserving region far from the source where Eq. (7) applies. Equations (8) and (9) were used to extrapolate present measurements of mean mixture fractions and velocities along the axis to find the corresponding values of the virtual origins, as discussed earlier.

Results and Discussion

Mean Velocities. The evolution of mean and fluctuating mixture fractions from source to self-preserving conditions has been considered by Dai et al. (1994). Present measurements of velocity properties were limited to the region where self-preserving behavior was observed for mean and fluctuating mixture fractions; namely, $(x-x_o)/d \geq 87$ and $(x-x_o)/\ell_M \geq 12$. Within this region, velocity properties also were observed to be self preserving. Present measurements of mean streamwise velocities for the self-preserving region are illustrated in Fig. 1. The scaling parameters of Eq. (8) are used in the figure so that the ordinate is equal to $U(r/(x-x_o))$. The variation of $U(r/(x-x_o))$ is seen to be universal within experimental

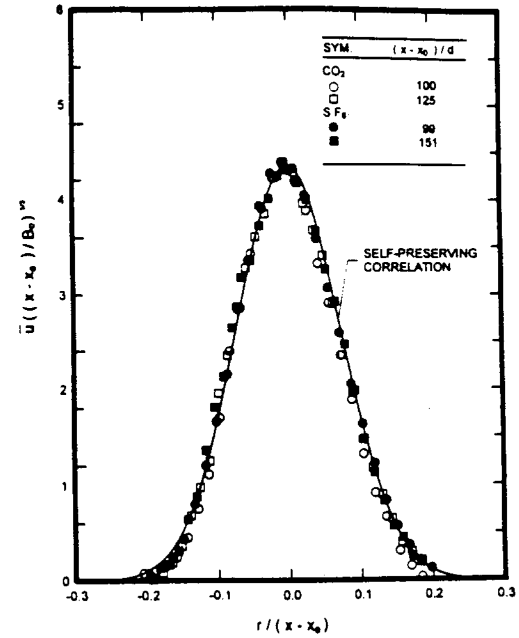

Fig. 1 Radial profiles of mean streamwise velocities in self-preserving buoyant turbulent plumes.

uncertainties over the range of the measurements, as anticipated for self-preserving flow. In contrast, results at smaller values of $(x-x_o)/d$, not shown in Fig. 1, exhibited broader profiles and smaller values of U(0), analogous to the behavior of $\bar{f}$ observed by Dai et al. (1994) in the transitional region of buoyant turbulent plumes.

Within the self-preserving region, present radial profiles of mean streamwise velocities are reasonably approximated by a Gaussian fit, similar to past work (Rouse et al., 1952; Papanicolaou and List, 1988; Ogino et al., 1980; Nakagome and Hirata, 1977; Shabbir and George, 1992; George et al., 1977) as follows:

$$U(r/(x-x_o)) = U(0)\exp\{-k_u^2(r/(x-x_o))^2\} \quad (11)$$

where

$$k_u = (x-x_o)/\ell_u \quad (12)$$

and ℓ_u is a characteristic plume radius where $\bar{u}/\bar{u}_c = \exp(-1)$. The best fit of the present data in the self-preserving region yielded U(0) = 4.3 and $k_u^2 = 93$, with $\ell_u/(x-x_o) = 0.10$. The resulting correlation is seen to be a good representation of the measurements illustrated in Fig. 1.

Present measurements of mean streamwise velocities in the self-preserving region of turbulent plumes yield narrower profiles with larger values near the axis (when appropriately scaled) than earlier results obtained at smaller distances from the source. This behavior is quantified in Table 2, where the range of streamwise distances considered for measurements of radial profiles of self-preserving plume properties, and the corresponding reported values of k_u^2, U(0) and $\ell_u/(x-x_o)$, are summarized for representative past studies and associated earlier work from the same laboratories. Past measurements generally satisfy the criterion for buoyancy-dominated flow, i.e., $(x-x_o)/\ell_M > 6$ (Papanicolaou and List, 1987, 1988). However, except for the present study, the measurements were obtained at values of $(x-x_o)/d$ that normally are not associated with self-preserving conditions for jet-like sources. Somewhat like the tendency for transitional plumes to have broader profiles than the self-preserving region, mentioned earlier, the values of k_u^2 tend to increase as the maximum streamwise position considered is increased, exhibiting an increase of 40% for the range of conditions given in Table 2. This yields a corresponding reduction of the characteristic plume radius of 40%, and an increase of U(0) of 25%, when approaching self-preserving conditions over the range considered in Table 2.

Present measurements of radial profiles of mean radial velocities are illustrated in Fig. 2. The scaling parameters used in the figure for $\bar{v}$ and radial distance provide universal plots within the self-preserving region as well as a check of the internal consistency of the present measurements of $\bar{u}$ and $\bar{v}$. This behavior can be seen from conservation of mass in the self-preserving region where the variation of density is small, e.g.:

$$r\partial \bar{u}/\partial x + \partial r\bar{v}/\partial r = 0 \tag{13}$$

Integrating Eq. (13), noting that $r\bar{u} = 0$ at $r=0$, then yields:

$$r\bar{v}/((x-x_o)\bar{u}_c) = \int_o^\eta ((x-x_o)/\bar{u}_c)(\partial \bar{u}/\partial x)\eta d\eta \tag{14}$$

where

$$\eta = r/(x-x_o) \tag{15}$$

The integral on the right-hand side of Eq. (14) can be evaluated for self-preserving conditions after substituting from Eqs. (8) and (11) for $\bar{u}$ and $\bar{u}_c$. This yields:

$$r\bar{v}/((x-x_o)\bar{u}_c) = (5/6k_u^2)[(1+6k_u^2\eta^2/5)\exp(-k_u^2\eta^2)-1] \tag{16}$$

which demonstrates the universality of the scaled value of $r\bar{v}$ as a function of η within the self-preserving region. Finally, adopting $k_u^2 = 93$ from present measurements (see Table 2) yields the plot based on the mean streamwise velocity measurements illustrated in Fig. 2.

The measurements of $\bar{v}$ illustrated in Fig. 2 exhibit universal behavior for the two test plumes, as anticipated for the self-preserving region. Additionally, the measurements of $\bar{v}$ also are consistent with present measurements of $\bar{u}$, based on good agreement with the correlation found from $\bar{u}$ through the continuity equation. This check is important because $\bar{v}$ is small, roughly an order of magnitude smaller than $\bar{v}'$, so that it is difficult to measure due to its rather low signal-to-noise ratios.

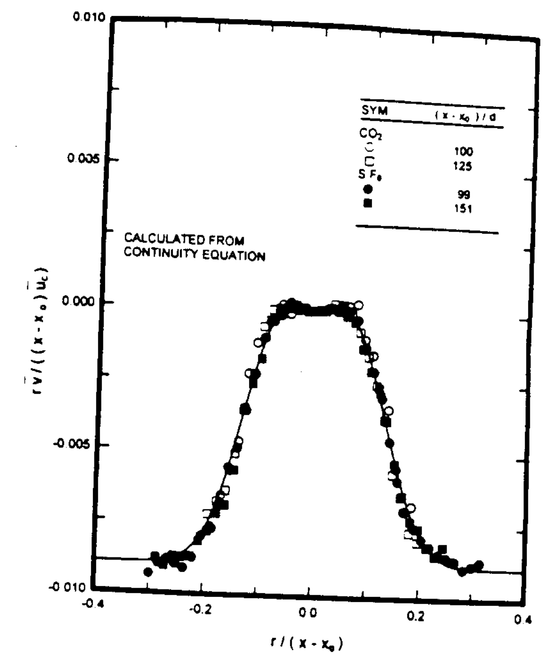

Fig. 2 Radial profiles of mean radial velocities in self-preserving buoyant turbulent plumes.

The asymptotic values of $r\bar{v}$ at large absolute values of η are proportional to the entrainment constant of the plumes, which is important for integral theories of plume scaling and as a measure of turbulent mixing rates (Morton, 1959; Morton et al., 1956). Entrainment behavior can be seen by integrating Eq. (13) to obtain an expression for the rate of change of the volumetric flow rate within the plumes at self-preserving conditions where the density of the flow is nearly constant:

$$d/dx\int_o^\infty r\bar{u}dr = dQ/dx = -(r\bar{v})_\infty \tag{17}$$

Table 2 Summary of self-preserving buoyant turbulent plume velocity properties[a]

Source	Medium	$(x-x_o)/d$	$(x-x_o)/\ell_M$	k_u^2	$\ell_u/(x-x_o)$	U(0)	$(\bar{u}'/\bar{u})_c$	E_o[b]
Present study	gaseous	87-151	12-43	93	0.10	4.3	0.22	0.086
Papanicolaou and List (1988)	liquid	22-62	9-62	90	0.11	3.9	0.25	0.088
Shabbir and George (1992)	gaseous	10-25	6-15	58	0.13	3.4	0.32	0.109
George et al. (1977)	gaseous	8-16	6-12	55	0.14	3.4	0.28	0.112
Ogino et al. (1980)	liquid	6-36	5-15	51	0.14	3.4	---	0.117
Nakagome and Hirata (1977)	gaseous	5-13	∞	48	0.14	3.9	0.25	0.120

[a]Round turbulent plumes in still, unstratified environments. Range of streamwise distances are for conditions where quoted self-preserving properties were found from measurements over the cross section of the plumes. Entries are ordered in terms of decreasing k_u.

[b]Found from Eq. (20).

An estimate of $(r\bar{v})_\infty$ can be found from the measurements of mean streamwise velocities through Eq. (16), as follows:

$$-(r\bar{v})_\infty / ((x-x_o)\bar{u}_c) = 5/(6k_u^2) \quad (18)$$

In view of the agreement between measurements of $r\bar{v}$ and Eq. (16), discussed earlier, Eq. (18) provides a reasonable estimate of entrainment properties. Then, noting that $k_u^2 = 93$, Eq. (18) yields $-(r\bar{v})_\infty / ((x-x_o)\bar{u}_c) = 0.0090$.

Actual entrainment constants have values that depend on the characteristic radius and velocity used in their definition (Morton, 1959). For present purposes, it is convenient to use ℓ_u and $\bar{u}_c$ as the radius and velocity scales so that the entrainment constant, E_o, is defined as follows:

$$dQ/dx = E_o \, \ell_u \, \bar{u}_c \quad (19)$$

Then, an expression for E_o can be found in terms of measurements of mean streamwise velocities from Eqs. (12), (17), (18) and (19), as follows:

$$E_o = 5/(6k_u) \quad (20)$$

Values of E_o found from Eq. (20) are summarized in Table 2 for various existing measurements. The main trend of the data is for E_o to decrease as self-preserving conditions are approached at large distances from the source, with E_o decreasing roughly 40% over the range of existing measurements.

<u>Velocity Fluctuations</u>. Radial profiles of streamwise, radial and tangential velocity fluctuations for the self-preserving region of the two sources, $(x-x_o)/d \geq 87$ and $(x-x_o)/\ell_M \geq 12$, are illustrated in Figs. 3-5. It is seen that over the range of streamwise distances considered, the profiles are universal within experimental uncertainties. Results at smaller distances (not shown in Figs. 3-5), however, exhibited broader profiles, analogous to the other mean and fluctuating properties within the transitional region of buoyant turbulent plumes. The magnitude of streamwise turbulence intensities near the axis, however, actually decreases slightly during the latter stages of development of the transitional plumes. This behavior appears to be due to the increase of mean velocities near the axis (when appropriately scaled) as the self-preserving region of the flow is approached. Thus, present estimates of $(\bar{u}'/\bar{u})_c = 0.22$ generally are lower than values in the range 0.25-0.32 observed earlier for transitional plumes, see Table 2.

The presence of the dip in streamwise velocity fluctuations near the axis for the self-preserving region, seen in Figs. 3-5, is similar to the behavior of nonbuoyant jets, see Papanicolaou and List (1988) and references cited therein, and is expected because turbulence production is reduced near the axis due to symmetry. In contrast, Dai et al. (1994) did not observe a corresponding dip near the axis for mixture fraction fluctuations, within self-preserving buoyant turbulent plumes, which they attribute to buoyancy/turbulence interactions because such dips are observed in nonbuoyant turbulent jets (Becker et al., 1967). Another unusual effect of buoyancy is that streamwise turbulence intensities near the axis of self-preserving plumes are slightly lower than for nonbuoyant jets, 0.22 in comparison to 0.25, see Papanicolaou and List (1988) for a discussion of existing turbulent jet data. Thus, buoyancy/turbulence interactions simultaneously act to reduce velocity fluctuation intensities, and to increase mixture fraction fluctuation intensities, near the axis of self-preserving turbulent plumes, in comparison to values found for nonbuoyant jets. Other properties of the velocity fluctuations in the self-preserving region of buoyant turbulent plumes are qualitatively similar to nonbuoyant turbulent jets (List, 1982). For example, $\bar{v}' \approx \bar{w}'$ throughout the self-preserving region, while the outer edge of the flow is nearly isotropic, e.g., $\bar{u}' \approx \bar{v}' \approx \bar{w}'$ for $r/(x-x_o) \geq 0.2$. In contrast, the flow exhibits greater anisotropy near the axis, e.g., $\bar{u}' \approx 1.25\ \bar{v}' \approx 1.25\ \bar{w}'$ for $r/(x-x_o) <$

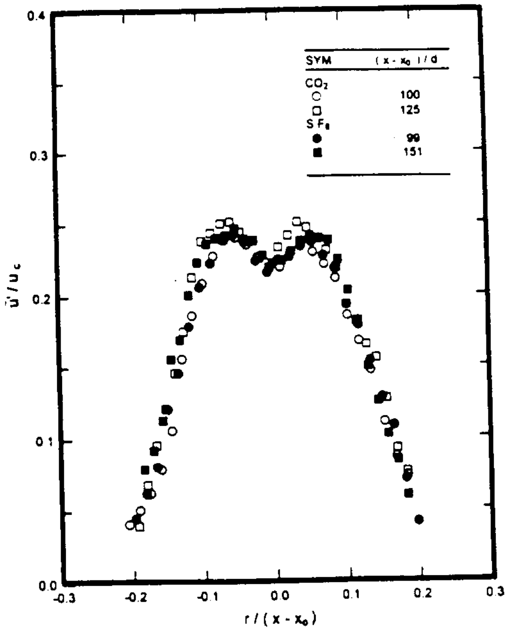

Fig. 3 Radial profiles of streamwise velocity fluctuations in self-preserving buoyant turbulent plumes.

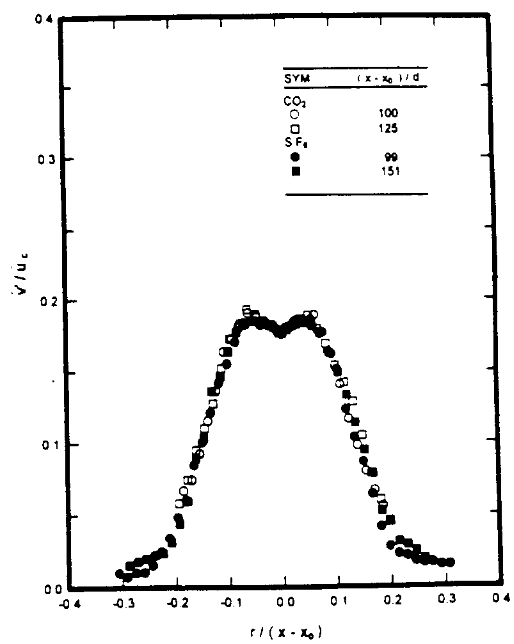

Fig. 4 Radial profiles of radial velocity fluctuations in self-preserving buoyant turbulent plumes.

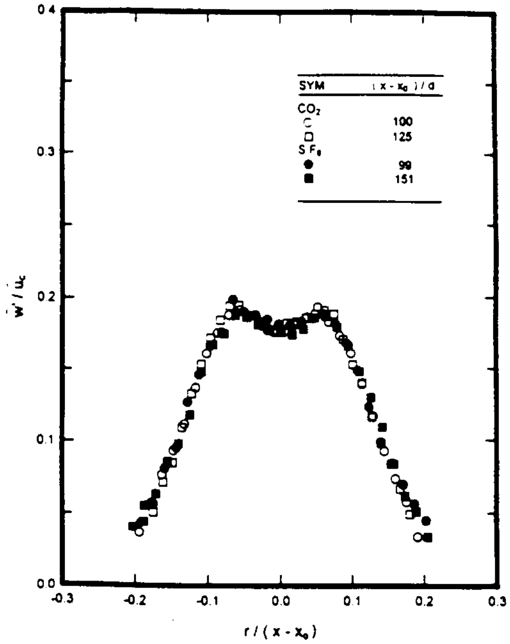

Fig. 5 Radial profiles of tangential velocity fluctuations in self-preserving buoyant turbulent plumes.

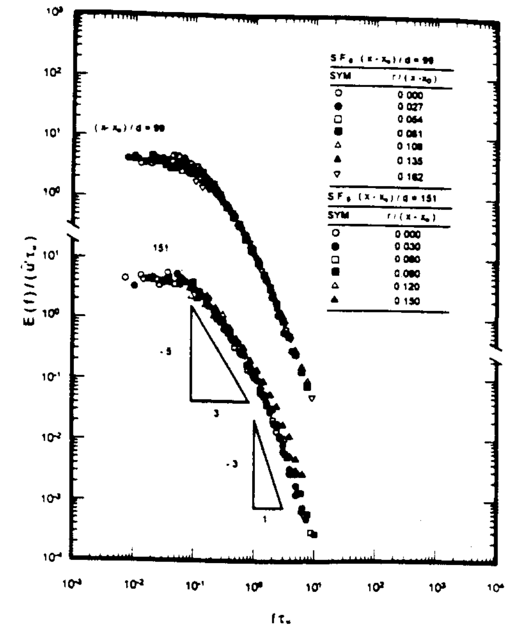

Fig. 6 Temporal power spectra of streamwise velocity fluctuations in the self-preserving portion of buoyant turbulent sulfur hexafluoride plumes.

0.1. Finally, the region of peak values of $\bar{u}'$, $\bar{v}'$ and $\bar{w}'$ is nearly the same, e.g., $r/(x-x_o)$ of roughly 0.05.

Some typical temporal power spectra of streamwise velocity fluctuations for the self-preserving region of the sulfur hexafluoride plumes are illustrated in Fig. 6. Results for the self-preserving region of the carbon dioxide plumes are similar. The spectra are relatively independent of radial position, when normalized in the manner of Fig. 6. The spectra initially decay according to the -5/3 power of frequency, analogous to the well known inertial-convection region for scalar property and velocity fluctuations in nonbuoyant turbulence (Tennekes and Lumley, 1972). At higher frequencies, however, there is a prominent region where the spectra decay according to the -3 power of frequency, which has been observed before but only in buoyant turbulent flows (Dai et al., 1994; Mizushima et al., 1979; Papanicolaou and List, 1987, 1988). This region has been called the inertial-diffusive subrange, where the variation of the local rate of dissipation of velocity fluctuations in buoyant flows is due to buoyancy-generated inertial forces rather than viscous forces. This effect is plausible due to the progressive increase in the span of the inertial range as $(x-x_o)/d$, or the plume Reynolds number, increases. For example, the intersections of the -5/3 and -3 subranges occur at higher values of $n\tau_u$ and $(x-x_o)/d$ increases; similar behavior has been observed for mixture fraction fluctuations in the self-preserving region of buoyant turbulent plumes (Dai et al., 1994). At higher frequencies, power spectral densities should become small as the Kolmogorov scale is approached. However, present measurements could not resolve this region due to seeding limitations which introduced effects of step noise.

The measured values of the temporal integral scales are illustrated in Fig. 7 for the self-preserving region of the two plumes. Additionally, streamwise spatial integral scales have been found from the temporal integral scale data using Taylor's hypothesis, e.g., $\Lambda_{ux} = \bar{u}\tau_u$, and are plotted in Fig. 7 as well. Self-preserving normalization has been used for both integral scales, similar to past treatment of integral scales relevant to mixture fraction fluctuations (Dai et al., 1994). Both integral scales approximate universal behavior for self-preserving conditions when plotted in the manner of Fig. 7. The spatial integral scale for streamwise velocity, Λ_{ux}, exhibits a slight reduction near the axis which is not observed for spatial integral scales associated with mixture fraction fluctuations. The increase of τ_u near the edge of the flow is similar to the behavior of τ_f observed by Dai et al. (1994); this increase is caused by smaller mean velocities near the edge of the flow, combined with relatively uniform spatial integral scales, through Taylor's hypothesis.

<u>Reynolds Stress</u>. Present measurements of Reynolds stress for the self-preserving region of the two plumes are illustrated in Fig. 8. The measurements are seen to exhibit universal behavior throughout the self-preserving region, with $\overline{u'v'} = 0$ at $r=0$, and then increasing to a maximum value near $r/(x-x_o) = 0.05$ (in the absolute sense), before decreasing to zero once again at large r. Notably, the region of the maximum Reynolds stress in Fig. 8 roughly corresponds to the region of maximum velocity fluctuations in Figs. 3-5.

The consistency of measured values of Reynolds stress with other measurements of mean and fluctuating quantities was evaluated similar to the conservation of mass considerations for $\bar{v}$, discussed earlier. Imposing the approximations of a thin, boundary-layer like plume flow, self-preserving conditions so that density variations are small, and neglecting viscous stresses in comparison to turbulent stresses, there results:

$$\bar{u}\partial\bar{u}/\partial x+\bar{v}\partial\bar{u}/\partial r = \partial/\partial x(\bar{v}'^2 - \bar{u}'^2+g(1-\rho_\infty/\rho_o)\bar{f} - \partial/\partial r(r\,\overline{u'v'})/r \quad (21)$$

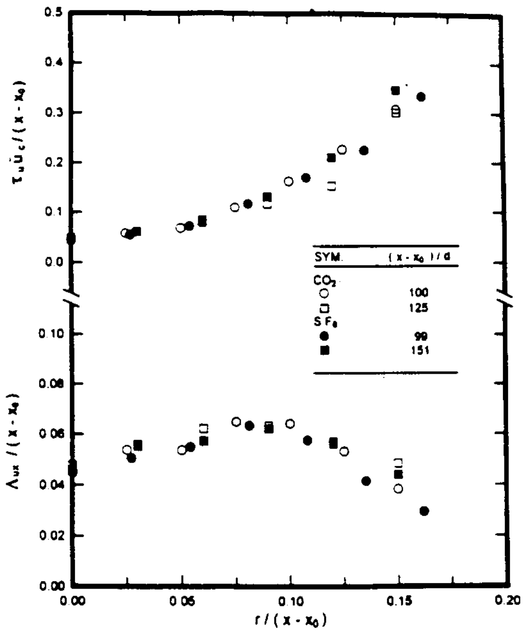

Fig. 7 Temporal and spatial integral scales of streamwise velocity fluctuations in self-preserving buoyant turbulent plumes.

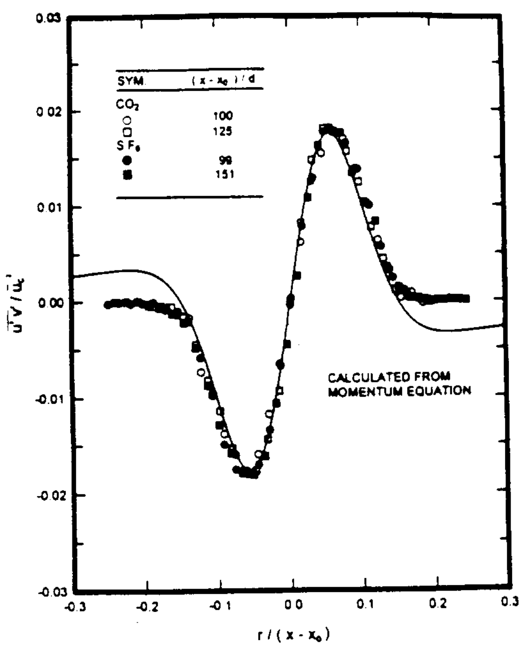

Fig. 8 Radial profiles of Reynolds stress in self-preserving buoyant turbulent plumes.

Then, integrating Eq. (21), assuming $(\bar{u}'^2 - \bar{v}'^2)/\bar{u}^2 << 1$, and using the present correlations for $\bar{u}$, and $\bar{v}$ and the correlation for $\bar{f}$ from Dai et al. (1994), all for the self-preserving portion of the flow, yields the following expression for Reynolds stresses:

$$\overline{u'v'}/\bar{u}_c^2 = (F(0)/(2\eta\, k_f^2 U(0)^2))(1-\exp(k_f^2\eta^2)) + (\eta + 1/(3\eta\, k_u^2))\exp(-2k_u^2\eta^2) - 1/(3k_u^2\eta)^2 - \bar{u}\,\bar{v}/\bar{u}_c^2 \quad (22)$$

Equation (22) also is plotted in Fig. 8. In general, this relationship is in good agreement with the measurements, implying reasonably good internal consistency of the present data. The main discrepancies between Eq. (22) and present measurements of Reynolds stresses are observed near the edge of the flow. This difficulty is mainly thought to be due to errors in the fits of mean velocities and mixture fractions by Gaussian functions near the edge of the flow.

Conclusions

Velocity statistics were measured in round buoyant turbulent plumes in still and unstratified air, in order to supplement earlier measurements of mixture fraction statistics for these flows due to Dai et al. (1994). The test conditions involved buoyant jet sources of carbon dioxide and sulfur hexafluoride to give ρ_o/ρ_∞ of 1.51 and 5.06 and source Froude numbers of 7.80 and 3.75, respectively, with $(x-x_o)/d$ in the range 87-151 and $(x-x_o)/\ell_M$ in the range 12-43. The major conclusions of the study are as follows:

1. Present measurements yielded distributions of mean streamwise velocities in self-preserving plumes that were up to 40% narrower, with larger mean streamwise velocities near the axis (when appropriately scaled), and smaller entrainment rates, than earlier results in the literature. The reason for these differences is that the earlier measurements were limited to $(x-x_o)/d \leq 62$ which is not a sufficient distance from the source to observe self-preserving behavior. Notably, these observations confirm the earlier results of Dai et al. (1994) concerning the requirements for self-preserving buoyant turbulent plume behavior, based on measurements of mixture fraction statistics.

2. Radial profiles of velocity fluctuations in self-preserving buoyant turbulent plumes and nonbuoyant turbulent jets are similar: the streamwise component exhibits a dip near the axis with intensities at the axis of roughly 22% for buoyant turbulent plumes, and there is a tendency toward isotropic behavior near the edge of the flow. This behavior is in sharp contrast to mixture fraction statistics where mixture fraction fluctuations within buoyant turbulent plumes do not exhibit a dip near the axis, unlike nonbuoyant turbulent jets. Effects of buoyancy/turbulence interactions causing this contrasting behavior for velocity and mixture fraction statistics clearly merit further study.

3. The temporal power spectra of streamwise velocity and mixture fraction fluctuations are qualitatively similar: the low frequency portions scale in a robust manner even in the transitional plume region, there is an inertial region where the spectra decay according to the -5/3 power of frequency, and there is an inertial-diffusive region at higher frequencies where the spectra decay according to the -3 power of frequency. The inertial-diffusive region has been observed by others for buoyant flows but is not observed in nonbuoyant flows; thus, the inertial-diffusive region is an interesting buoyancy/turbulence interaction that merits further study.

4. Past evaluations of turbulence models for buoyant turbulent flows, based on the assumption of self-preserving behavior for earlier measurements within buoyant turbulent plumes, should be reconsidered. In particular, present measurements suggest that such evaluations were compromised by effects of flow development because past measurements generally involved transitional plumes.

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